

Dimensional regularization of nonlinear sigma models on a finite time interval

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Abstract

We extend dimensional regularization to the case of compact spaces. Contrary to previous regularization schemes employed for nonlinear sigma models on a finite time interval (“quantum mechanical path integrals in curved space”) dimensional regularization requires only a covariant finite two-loop counterterm. This counterterm is nonvanishing and given by $\frac{1}{8}\hbar^2 R$.

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on a one dimensional worldline (quantum mechanical path integrals in curved space) has a long and confusing history. Early on, it was noticed that one obtains extra finite noncovariant counterterms of order \hbar^2 in the actions for the path integral if one goes from the hamiltonian to the lagrangian approach. These results were obtained in various ways: by using the Schroedinger equation for the transition element [1], Weyl ordering of the hamiltonian [2, 3], canonical point transformations in path integrals with time slicing [3] or by making a change of variables at the operatorial level from field variables to collective coordinates and nonzero modes [4]. Also in standard four dimensional gauge field theories such order \hbar^2 counterterms were found to be present if one chooses the Coulomb gauge [5, 6].

Having fixed the counterterms in the action for the path integral has no meaning by itself. One must also specify the regularization scheme. Nonlinear sigma models contain double derivative couplings so they are superficially divergent at the one- and two-loop levels by power counting. Different counterterms correspond to different regularization schemes. In fact, one first chooses a regularization scheme and then determines the corresponding counterterms.

The last decade two schemes were studied in detail [7, 8, 9, 10, 11, 12]: (i) mode regularization (MR) [7, 8] according to which the quantum fluctuations $q(\tau)$ around a background solution $x_{cl}(\tau)$ are expanded in a Fourier sine series cut-off at mode N and all calculations are performed before letting N tend to infinity, and (ii) time slicing (TS) [9] according to which only N variables $q(\tau_1), \dots, q(\tau_N)$ appear in the action at equally spaced points τ_i . In the latter case exact propagators were developed for finite N and the limit $N \rightarrow \infty$ could already be implemented in the Feynman rules themselves.

Of course, different regularization schemes give results which differ by finite local counterterms. In mode regularization these counterterms were fixed by requiring that the transition element $\langle x | \exp(-\frac{\tau}{\hbar} H) | y \rangle$ can also be obtained from a path integral with an action which differs from the naive action and which is fixed by requiring that the transition element satisfies the Schroedinger equation with the hamiltonian H . In time slicing one also obtains a path integral representation for $\langle x | \exp(-\frac{\tau}{\hbar} H) | y \rangle$ by inserting complete sets of position and momentum eigenstates, but here all steps are deductive and there is no need to impose the Schroedinger equation. Since Feynman graphs are regulated differently it comes as no surprise that also the counterterms are different. One finds

$$\begin{aligned} V_{MR} &= \frac{\hbar^2}{8} R - \frac{\hbar^2}{24} g^{ij} g^{kl} g_{mn} \Gamma_{ik}^m \Gamma_{jl}^n \\ V_{TS} &= \frac{\hbar^2}{8} R + \frac{\hbar^2}{8} g^{ij} \Gamma_{ik}^l \Gamma_{jl}^k. \end{aligned} \tag{1}$$

With these counterterms, both schemes give the same answers. In particular, a covariant

counterterms in the path integral to obtain the same covariant answer for the transition element. Numerous two- and three-loop calculations have confirmed these schemes [11, 12, 13]. Yet, it might simplify the calculations if a regularization scheme were found that only needs covariant counterterms. In fact, this has been a wish for workers in the field for quite a long time. One might think of using geodesic time slicing, but the positions of the intermediate points $q(\tau_1), \dots, q(\tau_N)$ would depend on the path considered and complexities overwhelm efforts in this direction.

The precise form of the counterterms has been a source of confusion in the past. Various coefficients for the counterterm with R have appeared in the literature. Kleinert has claimed that there are no counterterms needed at all for nonlinear sigma models, not even covariant counterterms [14]. To test these ideas he and Chervyakov have recently [15] studied a free particle in a box of length d by replacing the confining box by a smooth convex potential $V(x) = \frac{1}{2} \frac{m^2}{g} \tan^2(\sqrt{g}x)$ which grows to infinity near the walls ($x = \pm \frac{d}{2}$). The field redefinition $x \rightarrow \varphi = \frac{1}{\sqrt{g}} \tan(\sqrt{g}x)$ was made to obtain a nonlinear sigma model with a mass term $\frac{1}{2}m^2\varphi^2$. Using dimensional regularization it was found that both models gave the same results “so that there is no need for an artificial potential term of order \hbar^2 called for by previous authors” [15].

We disagree with the claim that no order \hbar^2 terms are needed in general: for this particular nonlinear sigma model the metric is flat so that R vanishes, but in general one needs a counterterm proportional to $\hbar^2 R$. However, noncovariant counterterms proportional to squares of the Christoffel symbols are apparently absent in this model if one uses dimensional regularization. This suggested to us to study dimensional regularization applied to general nonlinear sigma models. For an infinite time interval we indeed recently found that one needs only a covariant counterterm $\frac{1}{8}\hbar^2 R$ [16], but for massless nonlinear sigma models one must add by hand a noncovariant mass term $\frac{1}{2}m^2 x^2$ in order to regulate infrared divergences, and the result depends on m . For the really interesting applications (to anomalies and correlation functions of quantum field theories) one needs a finite time interval. In this case there are no infrared divergences and covariance can be maintained. In this letter we shall show that if one uses dimensional regularization for nonlinear sigma models on a finite time interval one needs only a covariant counterterm $V_{DR} = \frac{1}{8}\hbar^2 R$.

The model we consider is given by the following action

$$S[x^i] = \int_{-1}^0 d\tau \frac{1}{2} g_{ij}(x) \dot{x}^i \dot{x}^j. \quad (2)$$

Decomposing the paths $x^i(\tau)$ into a classical part $x_{cl}^i(\tau)$ satisfying suitable boundary conditions, and quantum fluctuations $q^i(\tau)$ which vanish at the boundary ($q^i(-1) = q^i(0) = 0$)

becomes formally

$$\begin{aligned} \langle 0|Tq^i(\tau)q^j(\sigma)|0\rangle &= -g^{ij}(0)\Delta(\tau, \sigma) \\ \Delta(\tau, \sigma) &= \sum_{n=1}^{\infty} \left[-\frac{2}{\pi^2 n^2} \sin(\pi n \tau) \sin(\pi n \sigma) \right] = \tau(\sigma + 1)\theta(\tau - \sigma) + \sigma(\tau + 1)\theta(\sigma - \tau). \end{aligned} \quad (3)$$

In MR one truncates the sum to N modes and sends $N \rightarrow \infty$ at the end of the calculations, while in TS one uses $\frac{\partial}{\partial \sigma} \theta(\sigma - \tau) = \delta(\sigma - \tau)$ where $\delta(\sigma - \tau)$ acts like a Kronecker delta, implying for example that $\iint \delta(\sigma - \tau) \theta(\sigma - \tau) \theta(\sigma - \tau) = \frac{1}{4}$ (and not equal to $\frac{1}{3}$ as one might perhaps naively expect from replacing the integrand with $\frac{1}{3} \frac{\partial}{\partial \sigma} \theta^3(\sigma - \tau)$). In general products of distributions are ambiguous, but going back to time slicing they are well defined). With these prescriptions one can unambiguously compute loop graphs.

To extend dimensional regularization to a compact time interval $-1 \leq \tau \leq 0$ we introduce D extra infinite dimensions $\mathbf{t} = (t^1, \dots, t^D)$, and take the limit $D \rightarrow 0$ at the end, as in standard dimensional regularization [17]. We also require translational invariance in the extra dimensions. As action in the $D + 1$ dimensions we take

$$S[x, a, b, c] = \int d^{D+1}t \left[\frac{1}{2} g_{ij}(x) (\partial_\mu x^i \partial^\mu x^j + a^i a^j + b^i c^j) + V_{DR}(x) \right] \quad (4)$$

where V_{DR} is the counterterm in dimensional regularization, $t^\mu = (\tau, \mathbf{t})$ with $\mu = 0, 1, \dots, D$ and $d^{D+1}t = d\tau d^D \mathbf{t}$. The propagators for this action read

$$\Delta(t, s) = \int \frac{d^D \mathbf{k}}{(2\pi)^D} \sum_{n=1}^{\infty} \frac{-2}{(\pi n)^2 + \mathbf{k}^2} \sin(\pi n \tau) \sin(\pi n \sigma) e^{i\mathbf{k} \cdot (\mathbf{t} - \mathbf{s})}. \quad (5)$$

The coordinates \mathbf{t} and \mathbf{s} for the extra D dimensions run from $-\infty$ to ∞ , and also the D continuous momenta \mathbf{k} run from $-\infty$ to ∞ . This propagator satisfies the Green equation⁴

$$(\partial_\tau^2 + \partial_{\mathbf{t}}^2) \Delta(t, s) = \delta^{D+1}(s, t) = \delta(\tau, \sigma) \delta^D(\mathbf{t} - \mathbf{s}) \quad (6)$$

where $\delta(\tau, \sigma) = \sum_{n=1}^{\infty} 2 \sin(\pi n \tau) \sin(\pi n \sigma)$ is the Dirac delta on the space of functions which vanish at $\tau, \sigma = -1, 0$.

In addition to the point particle coordinates $x^i(t)$ there are ghosts: one real commuting ghost $a^i(t)$ and two real anticommuting ghosts $b^i(t)$ and $c^i(t)$ [7, 8]. They appear in the action in the combination $\partial_\mu x^i(t) \partial^\mu x^j(t) + a^i(t) a^j(t) + b^i(t) c^j(t)$, and have propagators

$$\begin{aligned} \langle 0|T a^i(t) a^j(s)|0\rangle &= g^{ij}(0) \Delta_{gh}(t, s), \quad \langle 0|T b^i(t) c^j(s)|0\rangle = -2g^{ij}(0) \Delta_{gh}(t, s) \\ \Delta_{gh}(t, s) &= \int \frac{d^D \mathbf{k}}{(2\pi)^D} \sum_{n=1}^{\infty} 2 \sin(\pi n \tau) \sin(\pi n \sigma) e^{i\mathbf{k} \cdot (\mathbf{t} - \mathbf{s})} = \delta^{D+1}(t, s) = \delta(\tau, \sigma) \delta^D(\mathbf{t} - \mathbf{s}). \end{aligned} \quad (7)$$

⁴An alternative ansatz is suggested by writing the sines in (5) as $(\exp i\pi n(\tau - \sigma) - \exp i\pi n(\tau + \sigma))$ and modifying it into $(\exp i k_\mu(t^\mu - s^\mu) - \exp i k_\mu(t^\mu + s^\mu))$, where $k_\mu = (\pi n, \mathbf{k})$. This last expression has been used for finite temperature physics (J. Zinn-Justin, private communication, unpublished), but is not suitable for our purposes as it does not satisfy the Green equation.

tribute to higher loops in exactly the same way as ghosts in gauge theories. Although their propagators are formally equal to delta functions which vanish in standard dimensional regularization, they do contribute in our case because there are no infrared divergences, so that the usual cancellation between infrared and ultraviolet divergences in $\int d^{D+1}k = 0$ does not take place.

We can now calculate loop graphs treating the D dimensional momenta as in ordinary dimensional regularization, and performing the sums over n as in finite temperature physics. We compute all two-loops graphs which contribute to the vacuum energy. For this case we have $x_{cl}^i(\tau) = 0$. We shall give details of the calculations in an example below, but first summarize our result in the Table 1, where we give the results for each of the diagrams which contribute to the two-loop vacuum energy. In the last column we quote the tensor structure of the graphs with the shorthand notation $\partial^2 g \equiv g^{ij} g^{kl} \partial_k \partial_l g_{ij}$, $\partial^j g_j \equiv g^{ik} g^{jl} \partial_k \partial_l g_{ij}$, $\partial_k g \equiv g^{ij} \partial_k g_{ij}$ and $g_k \equiv g^{ij} \partial_i g_{jk}$. We record the results for time slicing, mode regularization and our version of dimensional regularization, respectively⁵.

It is clear that there are only differences for B_3 and B_4 . The computations in DR are done by using partial integration to bring all integrals in a form that can unambiguously be computed at $D \rightarrow 0$. The various manipulation are justified in dimensional regularization. In particular, partial integration is always allowed in the extra D dimension because of momentum conservation while it can be done in the finite time interval whenever there is an explicit function vanishing at the boundary (e.g. the propagator of the coordinates without derivatives). Let us use the notation $\frac{\partial}{\partial t^\mu} \Delta(t, s) = {}_\mu \Delta(t, s)$ and $\frac{\partial}{\partial s^\mu} \Delta(t, s) = \Delta_\mu(t, s)$ so that eq. (6) yields ${}_{\mu\mu} \Delta(t, s) = \Delta_{gh}(t, s) = \delta^{D+1}(t, s)$. The rule for contracting which indices with which indices follows from the action in (4). We find then for B_4 in dimensional regularization

$$\begin{aligned}
B_4(\text{DR}) &= \int_{-1}^0 d\tau \int_{-1}^0 d\sigma \quad \bullet \Delta (\Delta^\bullet) \bullet \Delta^\bullet \rightarrow \int d^{D+1}t \int d^{D+1}s \quad ({}_\mu \Delta) (\Delta_\nu) ({}_\mu \Delta_\nu) \\
&= \int d^{D+1}t \int d^{D+1}s \quad ({}_\mu \Delta) {}_\mu \left(\frac{1}{2} (\Delta_\nu)^2 \right) = -\frac{1}{2} \int d^{D+1}t \int d^{D+1}s \quad ({}_{\mu\mu} \Delta) (\Delta_\nu)^2 \\
&= -\frac{1}{2} \int d^{D+1}t \int d^{D+1}s \quad \delta^{D+1}(t, s) (\Delta_\nu)^2 = -\frac{1}{2} \int d^{D+1}t \quad (\Delta_\nu)^2|_\tau \\
&\rightarrow -\frac{1}{2} \int_{-1}^0 d\tau \quad \bullet \Delta^2|_\tau = -\frac{1}{24}
\end{aligned} \tag{8}$$

where the symbol $|_\tau$ means that one should set $\sigma = \tau$. Similarly

$$B_3(\text{DR}) = \int_{-1}^0 d\tau \int_{-1}^0 d\sigma \quad \Delta (\bullet \Delta^{\bullet 2} - \Delta_{gh}^2) \rightarrow \int d^{D+1}t \int d^{D+1}s \quad \Delta \left(({}_\mu \Delta_\nu) ({}_\mu \Delta_\nu) - ({}_{\mu\mu} \Delta) ({}_{\nu\nu} \Delta) \right) =$$

⁵To check the statement in the caption of Table 1 one may use that $R = \partial^2 g - \partial^j g_j - \frac{3}{4} (\partial_k g_{ij})^2 + \frac{1}{2} (\partial_i g_{jk}) \partial_j g_{ik} + \frac{1}{4} (\partial_j g)^2 - (\partial_j g) g^j + g_j^2$ and the $\Gamma\Gamma$ terms for TS are given by $-\frac{1}{8} \Gamma\Gamma = \frac{1}{32} (\partial_i g_{jk})^2 - \frac{1}{16} (\partial_i g_{jk}) (\partial_j g_{ik})$ while for MR one has $\frac{1}{24} \Gamma\Gamma = \frac{1}{32} (\partial_i g_{jk})^2 - \frac{1}{48} (\partial_i g_{jk}) (\partial_j g_{ik})$.

$$\begin{aligned}
& + \Delta (\mu \Delta) (\mu_{\nu\nu} \Delta) \Big) = \int d^{D+1}t \int d^{D+1}s \left(-(\mu \Delta) (\Delta_\nu) (\mu \Delta_\nu) + (\mu \Delta) (\mu \Delta) (\nu \nu \Delta) \right) \\
& = -B_4 + \int d^{D+1}t \int d^{D+1}s (\mu \Delta)^2 \delta^{D+1}(t, s) = -B_4 + \int d^{D+1}t (\mu \Delta)^2|_t = \frac{1}{8}.
\end{aligned} \tag{9}$$

We used the identity $(\mu\mu\Delta_\nu) = (\Delta_{\nu\mu\mu})$ obvious from (5). Moreover to compute diagrams like A_1 it is useful to use an identity which can be quickly derived in $D+1$ dimensions: recalling that we denote with a subscript 0 the derivative along the original compact time direction one has $((\mu\Delta_\mu)(t, s) + (\mu\mu\Delta)(t, s))|_{t=s} = {}_0[(\Delta(t, s))|_{t=s}]$.

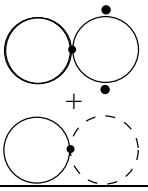
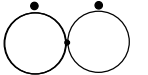
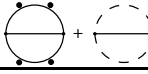

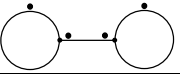
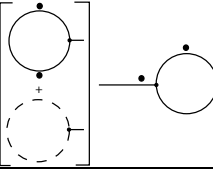
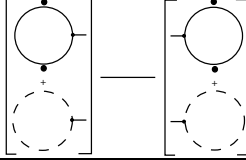
Integral	Results			Diagram	Tensor structure
	TS	MR	DR		
$A_1 \equiv \int \Delta _\tau (\bullet\Delta^\bullet + \Delta_{gh}) _\tau$	$-\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{1}{6}$		$-\frac{1}{4}\partial^2 g$
$A_2 \equiv \int (\bullet\Delta _\tau)^2$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$		$-\frac{1}{2}\partial^j g_j$
$B_3 \equiv \int \int \Delta (\bullet\Delta^{\bullet 2} - \Delta_{gh}^2)$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{8}$		$-\frac{1}{4}(\partial_i g_{jk})^2$
$B_4 \equiv \int \int (\bullet\Delta^\bullet) \Delta^\bullet (\bullet\Delta)$	$-\frac{1}{6}$	$-\frac{1}{12}$	$-\frac{1}{24}$		$-\frac{1}{2}(\partial_i g_{jk})\partial_j g_{ik}$
$B_5 \equiv \int \int \bullet\Delta _\tau (\bullet\Delta^\bullet) \Delta^\bullet _\sigma$	$-\frac{1}{12}$	$-\frac{1}{12}$	$-\frac{1}{12}$		$-\frac{1}{2}g_j^2$
$B_2 \equiv \int \int (\bullet\Delta^\bullet + \Delta_{gh}) _\tau \Delta^\bullet (\Delta^\bullet _\sigma)$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$		$-\frac{1}{2}(\partial_j g)g^j$
$B_1 \equiv \int \int (\bullet\Delta^\bullet + \Delta_{gh}) _\tau \Delta (\bullet\Delta^\bullet + \Delta_{gh}) _\sigma$	$-\frac{1}{12}$	$-\frac{1}{12}$	$-\frac{1}{12}$		$-\frac{1}{8}(\partial_j g)^2$

Table 1: 2-loop results with time slicing (TS), mode regularization (MR) and dimensional regularization (DR). Dots denote derivatives while hatched lined denote ghosts. For each scheme the sum of all graphs and the counterterms is $-\frac{1}{12}R$.

though one may question the following step

$$\int d^{D+1}t \int d^{D+1}s (\mu\Delta) (\Delta_\nu)^2 = \int d^{D+1}t (\Delta_\nu)^2|_t \quad (10)$$

where we used that formally $\mu\Delta(t, s) = \delta^{D+1}(t, s)$. The symbol $\delta^{D+1}(t, s)$ is an analytically continued delta function, and it is not clear that one may treat that as a regular delta function which is defined for D integer and positive. However, we recall that the correct prescription of dimensional regularization is to carry out all integrals over spacetime at integer dimensions before analytically continuing the momenta to D dimensions. Using this we can show by explicit calculation that (10) is correct. The right hand side of eq. (10) reads

$$\frac{1}{4} \int d\sigma d^D\mathbf{s} \sum_{m_1 \neq 0} \int \frac{d^D\mathbf{q}_1}{(2\pi)^D} \frac{(1 - e^{2\pi i m_1 \sigma})}{(\pi m_1)^2 + \mathbf{q}_1^2} \sum_{m_2 \neq 0} \int \frac{d^D\mathbf{q}_2}{(2\pi)^D} \frac{(1 - e^{2\pi i m_2 \sigma})}{(\pi m_2)^2 + \mathbf{q}_2^2} (-\mathbf{q}_1 \cdot \mathbf{q}_2 - \pi^2 m_1 m_2) I \quad (11)$$

where I is unity. The integral over \mathbf{s} gives the volume of the internal space which can be factored out, while the integrals over \mathbf{q}_1 and \mathbf{q}_2 are treated with ordinary dimensional regularization which makes the sums over m_1 and m_2 finite for sufficiently large negative D . Thus sums over modes of the finite time segment are made finite by dimensional regularization in the internal space. For the left hand side of equation (10), one obtains a similar result after extracting the exponents containing $(\tau - \sigma)$ from each of the three propagators, and performing the integrals over \mathbf{t} and τ , but now I is nontrivial

$$I = \frac{1}{2} \sum_{n \neq 0} (1 - e^{2\pi i n \sigma}) \left[\delta_{m_1+m_2+n,0} + \sum_l \delta_{m_1+m_2+n-(2l+1),0} \frac{2e^{-(2l+1)\pi i \sigma}}{i\pi(2l+1)} \right]. \quad (12)$$

We used that $\int_{-1}^0 d\tau e^{i\pi m \tau} = \delta_{m,0} + \sum_l \frac{2}{i\pi(2l+1)} \delta_{m-(2l+1),0}$ for any integer m . We can extend the sum over n to include $n = 0$. Performing the sum over n we obtain conditionally convergent series

$$I = \frac{1}{2} \left(1 - e^{-2\pi i(m_1+m_2)\sigma} \right) + \frac{1}{i\pi} \left(S(-\sigma) - e^{-2\pi i(m_1+m_2)\sigma} S(\sigma) \right) \quad (13)$$

where $S(\sigma) = \sum_l \frac{e^{(2l+1)i\pi\sigma}}{2l+1}$. The function $S(\sigma)$ is equal to $-\frac{i\pi}{2}$ for $-1 \leq \sigma \leq 0$, and $S(-\sigma) = -S(\sigma)$, hence I equals unity. This proves (10).

It may be useful to compare the calculations in DR with those using MR and TS. Consider the integral $\int_{-1}^0 \int_{-1}^0 (\bullet\Delta\bullet) (\Delta\bullet) (\bullet\Delta)$. In DR we wrote the integrand as $(\mu\Delta) (\Delta_\nu) (\mu\Delta_\nu) = (\mu\Delta) \mu(\frac{1}{2}(\Delta_\nu)^2)$, and partially integrated the second μ derivative to obtain

$$\begin{aligned} \int d^{D+1}t \int d^{D+1}s \left(-\frac{1}{2}\right)(\mu\Delta) (\Delta_\nu)^2 &= -\frac{1}{2} \int d^{D+1}t \int d^{D+1}s \delta^{D+1}(t, s) (\Delta_\nu)^2 = \\ &= -\frac{1}{2} \int d^{D+1}t (\Delta_\nu)^2|_t = -\frac{1}{24}. \end{aligned} \quad (14)$$

$\bullet\bullet\Delta = \delta(\tau - \sigma)$ because in MR both $\delta(\tau - \sigma)$ and $\theta(\tau - \sigma)$ in Δ^\bullet are smeared so that one would need to work out integrals of products of such MR regulated distributions. Instead we use the symmetry of $\bullet\bullet\Delta$ in τ and σ to replace $\bullet\bullet\Delta$ by $\Delta^{\bullet\bullet}$ and obtain then

$$B_4(\text{MR}) = -\frac{1}{2} \int_{-1}^0 d\tau \int_{-1}^0 d\sigma \frac{1}{3} \partial_\sigma (\Delta^\bullet)^3 = -\frac{1}{12} \quad (15)$$

where we used that $\Delta^\bullet = \tau + \theta(\sigma - \tau)$. In DR this procedure is not possible when μ is different from ν . In the TS scheme one obtains directly without partial integration

$$B_4(\text{TS}) = \int_{-1}^0 d\tau \int_{-1}^0 d\sigma [\tau + \theta(\sigma - \tau)][\sigma + \theta(\tau - \sigma)][1 - \delta(\tau - \sigma)] = -\frac{1}{6} \quad (16)$$

where we used that with TS one has $\int_{-1}^0 \int_{-1}^0 \theta(\tau - \sigma) \theta(\sigma - \tau) \delta(\tau - \sigma) = \frac{1}{4}$ while $\int_{-1}^0 \int_{-1}^0 \delta(\tau - \sigma) \tau \theta(\tau - \sigma) = -\frac{1}{4}$.

We have shown that one can regulate nonlinear sigma models on a finite time interval with an extension of dimensional regularization which keeps translational invariance in the extra D dimensions. Although the contributions to individual diagrams are different, we obtained the same results for the two-loop vacuum energy with this dimensional regularization as previously obtained with mode regularization and time slicing, but the new feature is that there are only covariant counterterms: $V_{DR} = \frac{\hbar^2}{8} R$. Presently, anomalies in higher dimensions are being calculated and we expect that our scheme will simplify such calculations. Also applications to scattering amplitudes may benefit from this scheme.

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